

3-8

Slopes of Parallel and Perpendicular Lines



Common Core State Standards

G-GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.

MP 1, MP 3, MP 4

Objective To relate slope to parallel and perpendicular lines

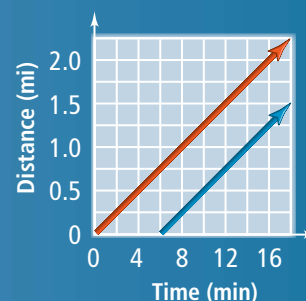


Think about what the slope of a line means in the context of this situation.



Getting Ready!

You and a friend enjoy exercising together. One day, you are about to go running when your friend receives a phone call. You decide to start running and tell your friend to catch up after the call. The red line represents you and the blue line represents your friend. Will your friend catch up? Explain.



In the Solve It, slope represents the running rate, or speed. According to the graph, you and your friend run at the same speed, so the slopes of the lines are the same. In this lesson, you will learn how to use slopes to determine how two lines relate graphically to each other.

Essential Understanding You can determine whether two lines are parallel or perpendicular by comparing their slopes.

When two lines are parallel, their slopes are the same.

Take note

Key Concept Slopes of Parallel Lines

- If two nonvertical lines are parallel, then their slopes are equal.
- If the slopes of two distinct nonvertical lines are equal, then the lines are parallel.
- Any two vertical lines or horizontal lines are parallel.

Think

Can you tell from the diagram whether the lines are parallel?

No. The lines may look parallel, but you only see a small portion of their graphs. Compare their slopes to know for sure.



Problem 1 Checking for Parallel Lines

Are lines ℓ_1 and ℓ_2 parallel? Explain.

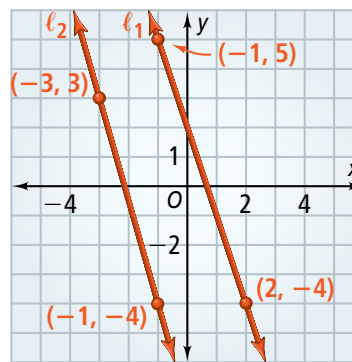
Step 1 Find the slope of each line.

$$\text{slope of } \ell_1 = \frac{5 - (-4)}{-1 - 2} = \frac{9}{-3} = -3$$

$$\text{slope of } \ell_2 = \frac{3 - (-4)}{-3 - (-1)} = \frac{7}{-2} = -\frac{7}{2}$$

Step 2 Compare the slopes.

Since $-3 \neq -\frac{7}{2}$, ℓ_1 and ℓ_2 are not parallel.



Got It? 1. Line ℓ_3 contains $A(-13, 6)$ and $B(-1, 2)$. Line ℓ_4 contains $C(3, 6)$ and $D(6, 7)$. Are ℓ_3 and ℓ_4 parallel? Explain.

Plan

How does the given line help you?

Parallel lines have the same slope. Once you know the slope of the given line, you know the slope you need to write an equation.



Problem 2 Writing Equations of Parallel Lines

What is an equation of the line parallel to $y = -3x - 5$ that contains $(-1, 8)$?

Think

Identify the slope of the given line.

You now know the slope of the new line and that it passes through $(-1, 8)$. Use point-slope form to write the equation.

Substitute -3 for m and $(-1, 8)$ for (x_1, y_1) and simplify.

Write

$$y = -3x - 5$$

$$y - y_1 = m(x - x_1)$$

$$\begin{aligned} y - 8 &= -3(x - (-1)) \\ y - 8 &= -3(x + 1) \end{aligned}$$



Got It? 2. What is an equation of the line parallel to $y = -x - 7$ that contains $(-5, 3)$?

When two lines are perpendicular, the product of their slopes is -1 . Numbers with product -1 are opposite reciprocals. This proof will be presented in more detail in Chapter 7.

Take note

Key Concept Slopes of Perpendicular Lines

- If two nonvertical lines are perpendicular, then the product of their slopes is -1 .
- If the slopes of two lines have a product of -1 , then the lines are perpendicular.
- Any horizontal line and vertical line are perpendicular.

Plan

Can you tell from the diagram whether the lines are perpendicular?

No. You can tell that the lines intersect, but not necessarily at right angles. So, you need to compare their slopes.



Problem 3 Checking for Perpendicular Lines

Lines ℓ_1 and ℓ_2 are neither horizontal nor vertical. Are they perpendicular? Explain.

Step 1 Find the slope of each line.

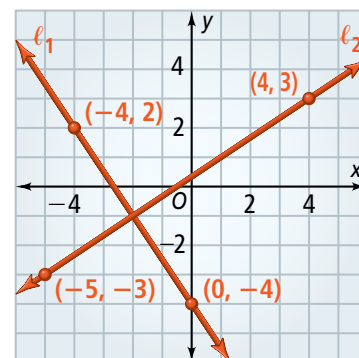
$$m_1 = \text{slope of } \ell_1 = \frac{2 - (-4)}{-4 - 0} = \frac{6}{-4} = -\frac{3}{2}$$

$$m_2 = \text{slope of } \ell_2 = \frac{3 - (-3)}{4 - (-5)} = \frac{6}{9} = \frac{2}{3}$$

Step 2 Find the product of the slopes.

$$m_1 \cdot m_2 = -\frac{3}{2} \cdot \frac{2}{3} = -1$$

Lines ℓ_1 and ℓ_2 are perpendicular because the product of their slopes is -1 .



Got It? 3. Line ℓ_3 contains $A(2, 7)$ and $B(3, -1)$. Line ℓ_4 contains $C(-2, 6)$ and $D(8, 7)$. Are ℓ_3 and ℓ_4 perpendicular? Explain.

Think

How is this similar to writing equations of parallel lines?

You follow the same process. The only difference here is that the slopes of perpendicular lines have product -1 .



Problem 4 Writing Equations of Perpendicular Lines

What is an equation of the line perpendicular to $y = \frac{1}{5}x + 2$ that contains $(15, -4)$?

Step 1 Identify the slope of the given line.

$$y = \frac{1}{5}x + 2$$

↑
slope

Step 2 Find the slope of the line perpendicular to the given line.

$$m_1 \cdot m_2 = -1 \quad \text{The product of the slopes of } \perp \text{ lines is } -1.$$

$$\frac{1}{5} \cdot m_2 = -1 \quad \text{Substitute } \frac{1}{5} \text{ for } m_1.$$

$$m_2 = -5 \quad \text{Multiply each side by 5.}$$

Step 3 Use point-slope form to write an equation of the new line.

$$y - y_1 = m(x - x_1)$$

$$y - (-4) = -5(x - 15) \quad \text{Substitute } -5 \text{ for } m \text{ and } (15, -4) \text{ for } (x_1, y_1).$$

$$y + 4 = -5(x - 15) \quad \text{Simplify.}$$

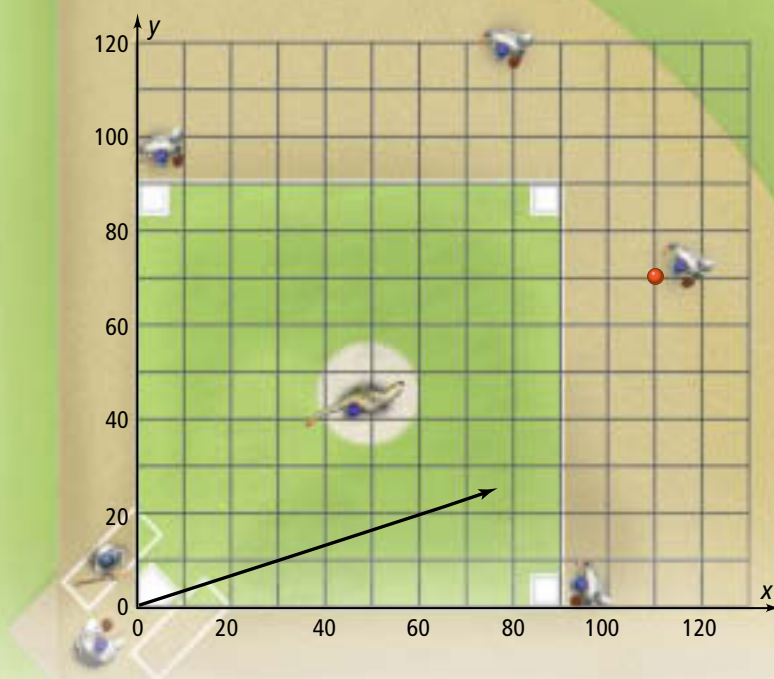


Got It? 4. What is an equation of the line perpendicular to $y = -3x - 5$ that contains $(-3, 7)$?



Problem 5 Writing Equations of Lines

Sports The baseball field below is on a coordinate grid with home plate at the origin. A batter hits a ground ball along the line shown. The player at $(110, 70)$ runs along a path perpendicular to the path of the baseball. What is an equation of the line on which the player runs?



Step 1 Find the slope of the baseball's path.

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20 - 10}{60 - 30} = \frac{10}{30} = \frac{1}{3}$$

Points $(30, 10)$ and $(60, 20)$ are on the baseball's path.

Step 2 Find the slope of a line perpendicular to the baseball's path.

$$m_1 \cdot m_2 = -1$$

The product of the slopes of \perp lines is -1 .

$$\frac{1}{3} \cdot m_2 = -1$$

Substitute $\frac{1}{3}$ for m_1 .

$$m_2 = -3$$

Multiply each side by 3.

Step 3 Write an equation of the line on which the player runs.

The slope is -3 and a point on the line is $(110, 70)$.

$$y - y_1 = m(x - x_1)$$

Point-slope form

$$y - 70 = -3(x - 110)$$

Substitute -3 for m and $(110, 70)$ for (x_1, y_1) .

Think

Which linear equation form should you use?

You know the slope. The player is located at a point on the line. Use point-slope form.



Got It? 5. Suppose a second player standing at $(90, 40)$ misses the ball, turns around, and runs on a path parallel to the baseball's path. What is an equation of the line representing this player's path?



Lesson Check

Do you know HOW?

\overleftrightarrow{AB} contains points A and B . \overleftrightarrow{CD} contains points C and D . Are \overleftrightarrow{AB} and \overleftrightarrow{CD} *parallel*, *perpendicular*, or *neither*? Explain.

1. $A(-8, 3)$, $B(-4, 11)$, $C(-1, 3)$, $D(1, 2)$
2. $A(3, 5)$, $B(2, -1)$, $C(7, -2)$, $D(10, 16)$
3. $A(3, 1)$, $B(4, 1)$, $C(5, 9)$, $D(2, 6)$
4. What is an equation of the line perpendicular to $y = -4x + 1$ that contains $(2, -3)$?

Do you UNDERSTAND?



MATHEMATICAL PRACTICES

- © 5. **Error Analysis** Your classmate tries to find an equation for a line parallel to $y = 3x - 5$ that contains $(-4, 2)$. What is your classmate's error?

$$\begin{aligned} \text{slope of given line} &= 3 \\ \text{slope of parallel line} &= \frac{1}{3} \\ y - y_1 &= m(x - x_1) \\ y - 2 &= \frac{1}{3}(x + 4) \end{aligned}$$

- © 6. **Compare and Contrast** What are the differences between the equations of parallel lines and the equations of perpendicular lines? Explain.



Practice and Problem-Solving Exercises

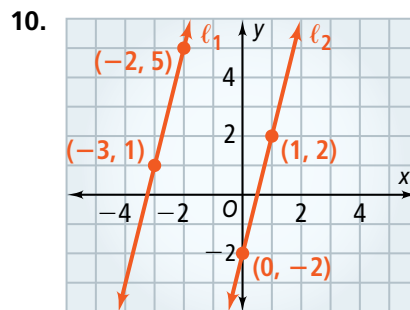
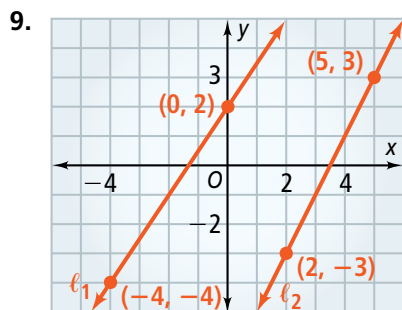
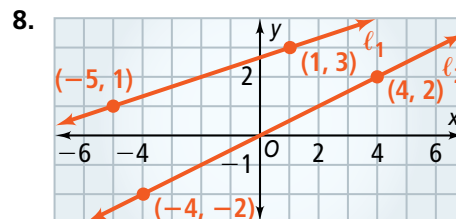
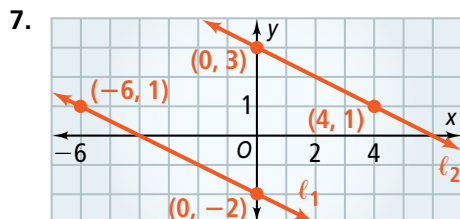


MATHEMATICAL PRACTICES

A Practice

For Exercises 7–10, are lines ℓ_1 and ℓ_2 parallel? Explain.

◀ See Problem 1.



Write an equation of the line parallel to the given line that contains C .

◀ See Problem 2.

11. $C(0, 3)$; $y = -2x + 1$

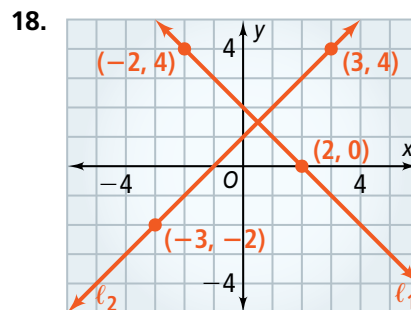
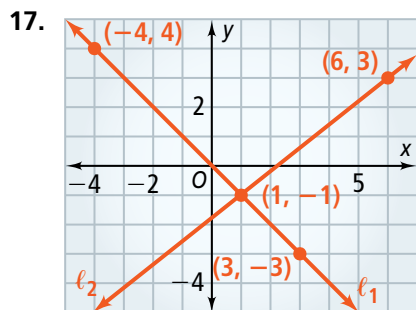
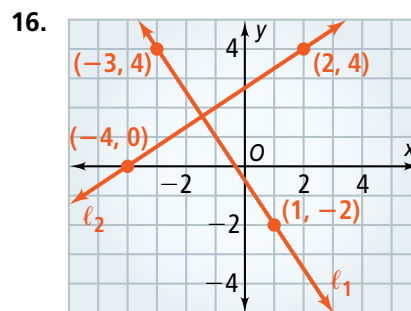
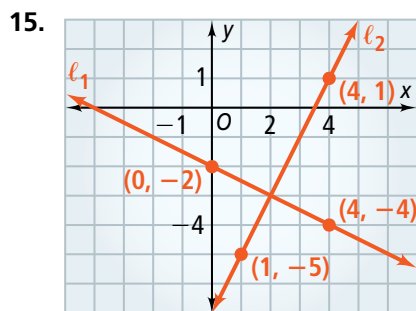
12. $C(6, 0)$; $y = \frac{1}{3}x$

13. $C(-2, 4)$; $y = \frac{1}{2}x + 2$

14. $C(6, -2)$; $y = -\frac{3}{2}x + 6$

For Exercises 15–18, are lines ℓ_1 and ℓ_2 perpendicular? Explain.

See Problem 3.



Write an equation of the line perpendicular to the given line that contains P .

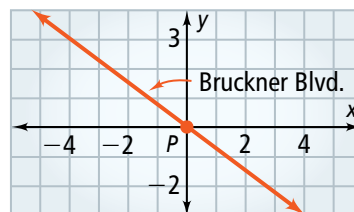
See Problem 4.

19. $P(6, 6); y = \frac{2}{3}x$

20. $P(4, 0); y = \frac{1}{2}x - 5$

21. $P(4, 4); y = -2x - 8$

- STEM** 22. **City Planning** City planners want to construct a bike path perpendicular to Bruckner Boulevard at point P . An equation of the Bruckner Boulevard line is $y = -\frac{3}{4}x$. Find an equation of the line for the bike path.



See Problem 5.



Rewrite each equation in slope-intercept form, if necessary. Then determine whether the lines are parallel. Explain.

23. $y = -x + 6$
 $x + y = 20$

24. $y - 7x = 6$
 $y + 7x = 8$

25. $3x + 4y = 12$
 $6x + 2y = 6$

26. $2x + 5y = -1$
 $10y = -4x - 20$

- © 27. **Think About a Plan** Line ℓ_1 contains $(-4, 1)$ and $(2, 5)$ and line ℓ_2 contains $(3, 0)$ and $(-3, k)$. What value of k makes ℓ_1 and ℓ_2 parallel?
- For ℓ_1 and ℓ_2 to be parallel, what must be true of their slopes?
 - What expressions represent the slopes of ℓ_1 and ℓ_2 ?
- © 28. **Open-Ended** Write equations for two perpendicular lines that have the same y -intercept and do not pass through the origin.
- © 29. **Writing** Can the y -intercepts of two nonvertical parallel lines be the same? Explain.

Use slopes to determine whether the opposite sides of quadrilateral $ABCD$ are parallel.

30. $A(0, 2), B(3, 4), C(2, 7), D(-1, 5)$

32. $A(1, 1), B(5, 3), C(7, 1), D(3, 0)$

31. $A(-3, 1), B(1, -2), C(0, -3), D(-4, 0)$

33. $A(1, 0), B(4, 0), C(3, -3), D(-1, -3)$

- © 34. **Reasoning** Are opposite sides of hexagon $RSTUVW$ at the right parallel? Justify your answer.

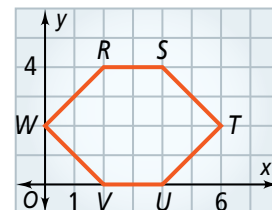
35. Which line is perpendicular to $3y + 2x = 12$?

(A) $6x - 4y = 24$

(B) $y + 3x = -2$

(C) $2x + 3y = 6$

(D) $y = -2x + 6$



Rewrite each equation in slope-intercept form, if necessary. Then determine whether the lines are perpendicular. Explain.

36. $y = -x - 7$

$y - x = 20$

37. $y = 3$

$x = -2$

38. $2x - 7y = -42$

$4y = -7x - 2$

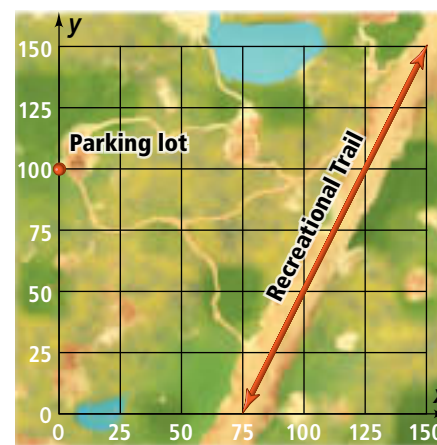
- © **Developing Proof** Explain why each theorem is true for three lines in the coordinate plane.

39. Theorem 3-7: If two lines are parallel to the same line, then they are parallel to each other.

40. Theorem 3-8: In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

41. **Rail Trail** A community recently converted an old railroad corridor into a recreational trail. The graph at the right shows a map of the trail on a coordinate grid. They plan to construct a path to connect the trail to a parking lot. The new path will be perpendicular to the recreational trail.

- Write an equation of the line representing the new path.
- What are the coordinates of the point at which the path will meet the recreational trail?
- If each grid space is 25 yd by 25 yd, how long is the path to the nearest yard?



- © 42. **Reasoning** Is a triangle with vertices $G(3, 2)$, $H(8, 5)$, and $K(0, 10)$ a right triangle? Justify your answer.

43. **Graphing Calculator** \overleftrightarrow{AB} contains points $A(-3, 2)$ and $B(5, 1)$. \overleftrightarrow{CD} contains points $C(2, 7)$ and $D(1, -1)$. Use your graphing calculator to find the slope of \overleftrightarrow{AB} . Enter the x-coordinates of A and B into the L1 list of your list editor. Enter the y-coordinates into the L2 list. In your **(stat)** **CALC** menu select **LinReg (ax + b)**. Press **(enter)** to find the slope a . Repeat to find the slope of \overleftrightarrow{CD} . Are \overleftrightarrow{AB} and \overleftrightarrow{CD} parallel, perpendicular, or neither?

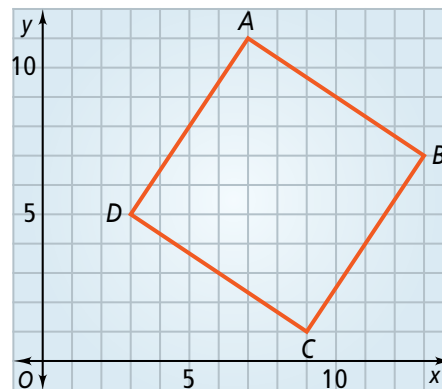
Challenge

For Exercises 44 and 45, use the graph at the right.

44. Show that the diagonals of the figure are congruent.
45. Show that the diagonals of the figure are perpendicular bisectors of each other.

46. a. Graph the points $P(2, 2)$, $Q(7, 4)$, and $R(3, 5)$.
b. Find the coordinates of a point S that, along with points P , Q , and R , will form the vertices of a quadrilateral with opposite sides parallel. Graph the quadrilateral.
c. Repeat part (b) to find a different point S . Graph the new quadrilateral.

47. **Algebra** A triangle has vertices $L(-5, 6)$, $M(-2, -3)$, and $N(4, 5)$. Write an equation for the line perpendicular to \overline{LM} that contains point N .

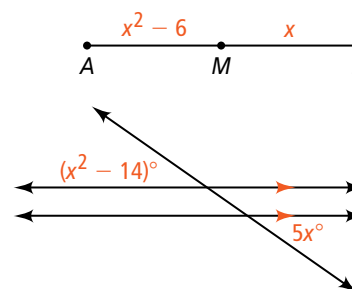


Standardized Test Prep

GRIDDED RESPONSE

SAT/ACT

48. $\triangle ABC$ is right with right angle C . The slope of \overline{AC} is -2 . What is the slope of \overline{BC} ?
49. In the diagram at the right, M is the midpoint of \overline{AB} . What is AB ?
50. What is the distance between $(-4.5, 1.2)$ and $(3.5, -2.8)$ to the nearest tenth?
51. What is the value of x in the diagram at the right?
52. The perimeter of a square is 20 ft. What is the area of the square in square feet?



Mixed Review

Algebra Write an equation for the line containing the given points.

53. $A(0, 3)$, $B(6, 0)$

54. $C(-4, 2)$, $D(-1, 7)$

55. $E(3, -2)$, $F(-5, -8)$

See Lesson 3-7.

Name the property that justifies each statement.

56. $\angle 4 \cong \angle 4$

57. If $m\angle B = 8$, then $2m\angle B = 16$.

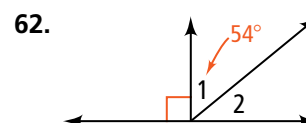
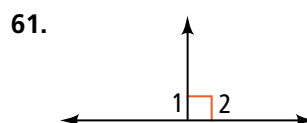
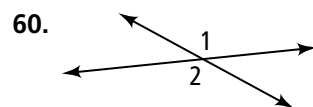
58. $-3x + 6 = 3(-x + 2)$

59. If $\overline{RS} \cong \overline{MN}$, then $\overline{MN} \cong \overline{RS}$.

See Lesson 2-5.

Get Ready! To prepare for Lesson 4-1, do Exercises 60–62.

Are $\angle 1$ and $\angle 2$ congruent? Explain.



See Lessons 1-5 and 2-6.